



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

Intrat.

Exit.

| Depositum | scrs | Depositum | scrs | Deposit. |
|-----------|------|-----------|----------|---|
| $n+1$ | 0 | Z | $n+1+p$ | 1 H $n+1$ A=Z |
| $n+1+p$ | 1 | Y | $n+1+2p$ | 2 K $n+1+p$ C=Y |
| $n+1+2p$ | 2 | X | $n+1+3p$ | 3 L $n+1+2p$ D= $\frac{1}{2}X+\frac{1}{2}\times\overline{Y+yp}$ |
| $n+1+3p$ | 3 | V | $n+1+4p$ | 4 M $n+1+3p$ E= $\frac{1}{2}V+\frac{1}{2}\times\overline{X+xp}+\frac{1}{2}\times\overline{Y+2yp}$ |
| $n+1+4p$ | 4 | T | | $n+1+4p$ F= $\frac{1}{2}T+\frac{1}{2}\times\overline{V+up}+\frac{1}{2}\times\overline{X+2xp}+\frac{1}{2}\times\overline{Y+3yp}$ |

N.º 2

$$\begin{aligned}
 &= \frac{1}{2} \times \overline{H-p} + \frac{1}{4} \times \overline{H-p+hp} + \frac{1}{8} \times \overline{H-p+2hp} + \frac{1}{16} \times \overline{H-p+3hp} + \dots + \frac{1}{2^n} \times \overline{H-p+np} - \overline{hp} + \frac{1}{2^n} \times \overline{np+n} + \dots \\
 &= \frac{1}{2} \times \overline{K-p} + \frac{1}{4} \times \overline{H-p+2hp} + \frac{1}{8} \times \overline{H-p+3hp} + \frac{1}{16} \times \overline{H-p+4hp} + \dots + \frac{1}{2^n} \times \overline{H-p+nhp} + \frac{1}{2^n} \times \overline{np+p} + \overline{n+1} \\
 &= \frac{1}{2} \times \overline{L-p} + \frac{1}{4} \times \overline{H-p+3hp} + \frac{1}{8} \times \overline{H-p+4hp} + \frac{1}{16} \times \overline{H-p+5hp} + \dots + \frac{1}{2^n} \times \overline{H-p+nhp} + \overline{hp} + \frac{1}{2^n} \times \overline{np+2p} + \overline{n+1} \\
 &= \frac{1}{2} \times \overline{M-p} + \frac{1}{4} \times \overline{H-p+4hp} + \frac{1}{8} \times \overline{H-p+5hp} + \frac{1}{16} \times \overline{H-p+6hp} + \dots + \frac{1}{2^n} \times \overline{H-p+nhp} + \overline{hp} + \frac{1}{2^n} \times \overline{np+3p} + \overline{n+1}
 \end{aligned}$$

N.º 3

$$\begin{aligned}
 &= \frac{1}{2^{n-1}} \times \overline{C+ncp-cp} + \frac{1}{2^{n-2}} \times \overline{D+ndp-2dp} + \frac{1}{2^{n-3}} \times \overline{E+ncp-3cp} + \frac{1}{2^{n-4}} \times \overline{F+ncp-4fp} + \dots \\
 &= \frac{1}{2^{n-3}} \times \overline{D+ndp-dp} + \frac{1}{2^{n-3}} \times \overline{E+ncp-2cp} + \frac{1}{2^{n-4}} \times \overline{F+ncp-3fp} + \dots \\
 &= \frac{1}{2^{n-3}} \times \overline{E+ncp-cp} + \frac{1}{2^{n-4}} \times \overline{F+ncp-2fp} + \dots \\
 &= \frac{1}{2^{n-4}} \times \overline{F+ncp-fp} + \dots
 \end{aligned}$$

N.º 4

N.º 6

N.º 8

$$\begin{aligned}
 -Z &= \frac{1}{2}K - \frac{1}{2}H + \frac{1}{4}hp + \frac{1}{8}hp + \frac{1}{16}hp + \dots + \frac{1}{2^n} \times hp = \frac{1}{2}K - \frac{1}{2}H + zp - \frac{1}{2}hp = -\frac{1}{2^{n-1}} \times C - \frac{ncp}{2^n} + zp \\
 -Y &= \frac{1}{2}L - \frac{1}{2}K + \frac{1}{4}hp + \frac{1}{8}hp + \frac{1}{16}hp + \dots + \frac{1}{2^n} \times hp = \frac{1}{2}L - \frac{1}{2}K + zp - \frac{1}{2}hp = -\frac{1}{2^{n-1}} \times D - \frac{ndp}{2^{n-1}} - \frac{cp}{2^n} + zp \\
 -X &= \frac{1}{2}M - \frac{1}{2}L + \frac{1}{4}hp + \frac{1}{8}hp + \frac{1}{16}hp + \dots + \frac{1}{2^n} \times hp = \frac{1}{2}M - \frac{1}{2}L + zp - \frac{1}{2}hp = -\frac{1}{2^{n-2}} \times E - \frac{ncp}{2^{n-2}} - \frac{dp}{2^{n-1}} - \frac{cp}{2^n} + zp
 \end{aligned}$$

N.º 5

N.º 7

$$\begin{aligned}
 -H &= -\frac{1}{2^{n-1}} \times \overline{C+ncp-cp} + \frac{1}{2^{n-2}} \times \overline{dp} + \frac{1}{2^{n-3}} \times \overline{cp} + \frac{1}{2^{n-4}} \times \overline{fp} + \dots = -\frac{1}{2^{n-1}} \times C - \frac{ncp}{2^{n-1}} + hp \\
 -K &= -\frac{1}{2^{n-2}} \times \overline{D+ndp-dp} + \frac{1}{2^{n-3}} \times \overline{cp} + \frac{1}{2^{n-4}} \times \overline{fp} + \dots = -\frac{1}{2^{n-2}} \times D - \frac{ndp}{2^{n-2}} - \frac{cp}{2^{n-1}} + hp \\
 -L &= -\frac{1}{2^{n-3}} \times \overline{E+ncp-cp} + \frac{1}{2^{n-4}} \times \overline{fp} + \dots = -\frac{1}{2^{n-3}} \times E - \frac{ncp}{2^{n-3}} - \frac{dp}{2^{n-2}} - \frac{fp}{2^{n-1}} + hp
 \end{aligned}$$

N.º 9.

N.º 10

N.º 8

$$\begin{aligned}
 Z &= A \\
 C &= C \\
 C &= 2D - C - cp \\
 C &= 4E - 2D - C - 2dp - 2cp \\
 C - A &= Y - Z = -\frac{1}{2^n} \times C - \frac{ncp}{2^n} + zp \\
 2D - 2C - cp &= X - Y = -\frac{1}{2^{n-1}} \times D - \frac{ndp}{2^{n-1}} - \frac{cp}{2^n} + zp \\
 4E - 4D - 2dp - cp &= V - X = -\frac{1}{2^{n-2}} \times E - \frac{ncp}{2^{n-2}} - \frac{dp}{2^{n-1}} - \frac{cp}{2^n} + zp
 \end{aligned}$$

N.º 11

N.º 12

$$\begin{aligned}
 Z &= \frac{A \times 2^n + ap \times 2^n - ncp}{1 + 2^n} \\
 C &= \frac{C \times 2^n + cp \times 2^{n-1} - \frac{1}{2} + ap \times 2^{n-1} - ndp}{1 + 2^n} \\
 V &= \frac{D \times 2^n + dp \times 2^{n-1} - \frac{1}{2} + cp \times 2^{n-2} - \frac{1}{4} + ap \times 2^{n-2} - ncp}{1 + 2^n} \\
 &= \frac{A + ap \times 2^n - ncp}{1 + 2^n} \\
 &= \frac{C + cp \times 2^n - ndp}{1 + 2^n} \\
 &= \frac{D + dp \times 2^n - ncp}{1 + 2^n}
 \end{aligned}$$

L. Senex sculp.

III. *Solutio generalis altera præcedentis Problematis, ope Combinati-
onum & Serierum infinitarum, per D. Abr. de Moivre. Reg.
Soc Sodalem.*

Designationes.

SI B & C collutores duo simul certent, ad designandum B victorem esse, C victum, scribatur BC ; atque vicissim ad designandum C victorem esse, B victum; scribatur CB : & sic de cæteris.

Ponatur 1° B vincere A , certamenque concludi tribus ludis

| | | |
|-----------------|---|--|
| $\frac{BA}{BC}$ | } | Sic patet B victorem necessario evadere. |
| BD | | |

Ponatur 2° B vincere A , certamenque concludi quatuor ludis

| | | |
|-----------------|---|--|
| $\frac{BA}{CB}$ | } | Sic patet C victorem necessario evadere. |
| CD | | |
| CA | | |

Ponatur 3° B vincere A , certamenque concludi quinque ludis

| | | | |
|-------------------|-----------------|---|---|
| $\frac{BA}{CB^*}$ | $\frac{BA}{BC}$ | } | Sic patet D victorem necessario evadere, id- que duplici modo. |
| DC | DB | | |
| DA | DA | | |
| DB | DC | | |
| | | | |

Ponatur 4° B prima vice vincere A , certamenque concludi sex ludis.

| | | |
|-------------------|-------------------|-----------------|
| \overline{BA} | \overline{BA} | \overline{BA} |
| \overline{CB} | $\overline{CB^*}$ | \overline{BC} |
| $\overline{DC^*}$ | $\overline{CD^*}$ | \overline{DB} |
| \overline{AD} | \overline{AC} | \overline{AC} |
| \overline{AB} | \overline{AB} | \overline{AD} |
| \overline{AC} | \overline{AD} | \overline{AB} |

Sic patet *A* victorem necessario evadere, idque triplici modo.

Ponatur 5^o certamen concludi septem ludis, ponaturque semper *B* prima vice vincere ipsum *A*.

| | | | | |
|-------------------|-------------------|-------------------|-------------------|-----------------|
| \overline{BA} | \overline{BA} | \overline{BA} | \overline{BA} | \overline{BA} |
| \overline{CB} | \overline{CB} | $\overline{CB^*}$ | \overline{BC} | \overline{BC} |
| \overline{DC} | $\overline{DC^*}$ | \overline{CD} | \overline{DB} | \overline{DB} |
| $\overline{AD^*}$ | \overline{DA} | \overline{AC} | $\overline{AD^*}$ | \overline{DA} |
| \overline{BA} | \overline{BD} | \overline{BA} | \overline{CA} | \overline{CD} |
| \overline{BC} | \overline{BC} | \overline{BD} | \overline{CB} | \overline{CB} |
| \overline{BD} | \overline{BA} | \overline{BC} | \overline{CD} | \overline{CA} |

Sic patet *B* vel *C* necessario victores evadere, *B* triplici modo, *C* duplici.

Ponatur 6^o certamen concludi octo ludis,

| | | | | | | | |
|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-----------------|
| \overline{BA} | \overline{BA} | \overline{BA} | \overline{BA} | \overline{BA} | \overline{BA} | \overline{BA} | \overline{BA} |
| \overline{CB} | \overline{CB} | \overline{CB} | \overline{CB} | $\overline{CB^*}$ | \overline{BC} | \overline{BC} | \overline{BC} |
| \overline{DC} | \overline{DC} | $\overline{DC^*}$ | \overline{CD} | \overline{CD} | \overline{DB} | \overline{DB} | \overline{DB} |
| \overline{AD} | $\overline{AD^*}$ | \overline{DA} | \overline{AC} | \overline{AC} | \overline{AD} | $\overline{AD^*}$ | \overline{DA} |
| $\overline{BA^*}$ | \overline{AB} | \overline{BD} | $\overline{BA^*}$ | \overline{AB} | $\overline{CA^*}$ | \overline{AC} | \overline{CD} |
| \overline{CB} | \overline{CA} | \overline{CB} | \overline{DB} | \overline{DA} | \overline{BC} | \overline{BA} | \overline{BC} |
| \overline{CD} | \overline{CD} | \overline{CA} | \overline{DC} | \overline{DB} | \overline{BD} | \overline{BD} | \overline{BA} |
| \overline{CA} | \overline{CB} | \overline{CD} | \overline{DA} | \overline{DC} | \overline{BA} | \overline{BC} | \overline{BD} |

Sic patet *C* victorem evadere triplici, *D* duplici, *B* triplici modo, &c.

Nunc ordine scribantur literæ quibus victores designantur.

| | |
|-----|---|
| 3, | 1 <i>B</i> |
| 4, | 1 <i>C</i> |
| 5, | 2 <i>D</i> |
| 6, | 3 <i>A</i> |
| 7, | 3 <i>B</i> + 2 <i>C</i> |
| 8, | 3 <i>C</i> + 2 <i>D</i> + 3 <i>B</i> . |
| 9, | 3 <i>D</i> + 2 <i>A</i> + 3 <i>C</i> + 3 <i>D</i> + 2 <i>A</i> |
| 10, | 3 <i>A</i> + 2 <i>B</i> + 3 <i>D</i> 3 <i>A</i> + 2 <i>B</i> + 3 <i>A</i> + 2 <i>C</i> + 3 <i>D</i> |

&c.

Per.

Perſpecta illarum formatione, patebit 1° literam *B* in ordine aliquo ſemper toties reperiri, quoties *A* in ordine ultimo & penultimo reperitur : 2° *C* in ordine aliquo toties reperiri quoties *B* in ordine ultimo & *D* in penultimo reperiuntur : 3° *D* in ordine aliquo toties reperiri quoties *C* in ultimo & *B* in penultimo : 4° *A* in ordine aliquo ſemper toties reperiri quoties *D* in ordine ultimo & *C* in penultimo reperiuntur.

Sed numerus variationum dato cuilibet ludorum numero competens, duplus eſt numeri variationum omnium dato ludorum numero unitate diminuto competentis : adeoque Probabilitas quam habet Colluſor *B* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *A* ut vinceret dato ludorum numero minus uno ; atque etiam ſubquadrupla probabilitatis quam habebat idem *A*, ut vinceret dato ludorum numero minus duobus : & ſic de cæteris.

Probabilitas quam habet *C*, ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *B*, ut vinceret dato ludorum numero minus uno ; atque etiam ſubquadrupla probabilitatis quam habebat *D*, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet *D* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *C*, ut vinceret dato ludorum numero minus uno ; atque etiam ſubquadrupla probabilitatis quam habebat *B*, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet *A* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *D*, ut vinceret dato ludorum numero minus uno ; atque etiam ſubquadrupla probabilitatis quam habebat *C* ut vinceret dato ludorum numero minus duobus.

Ex jam obſervatis facile eſt componere Tabulam Probabilitatum, quas *B*, *C*, *D*, *A* habent ut victores evadant dato ludorum numero, atque etiam illorum ſortium ſeu expectationum:

Tabula Probabilitatum, &c.

| | B | C | D | A |
|------|-------------------------------------|----------------------------------|----------------------------------|---------------------------------|
| ' | $3 \frac{1}{4} \times 4 + 3p$ | — | — | — |
| " | 4 — | $\frac{1}{8} \times 4 + 4p$ | — | — |
| ''' | 5 — | — | $\frac{2}{16} \times 4 + 5p$ | — |
| ''' | 6 — | — | — | $\frac{3}{32} \times 4 + 6p$ |
| v | $7 \frac{3}{64} \times 4 + 7p$ | $\frac{2}{64} \times 4 + 7p$ | — | — |
| v' | $8 \frac{3}{128} \times 4 + 8p$ | $\frac{3}{128} \times 4 + 8p$ | $\frac{2}{128} \times 4 + 8p$ | — |
| v'' | 9 — | $\frac{3}{256} \times 4 + 9p$ | $\frac{6}{256} \times 4 + 9p$ | $\frac{4}{256} \times 4 + 9p$ |
| v''' | $10 \frac{4}{512} \times 4 + 10p$ | $\frac{2}{512} \times 4 + 10p$ | $\frac{6}{512} \times 4 + 10p$ | $\frac{9}{512} \times 4 + 10p$ |
| 'x | $11 \frac{13}{1024} \times 4 + 11p$ | $\frac{10}{1024} \times 4 + 11p$ | $\frac{2}{1024} \times 4 + 11p$ | $\frac{9}{1024} \times 4 + 11p$ |
| x | $12 \frac{18}{2048} \times 4 + 12p$ | $\frac{19}{2048} \times 4 + 12p$ | $\frac{14}{1048} \times 4 + 12p$ | $\frac{4}{2048} \times 4 + 12p$ |
| | <i>&c.</i> | <i>&c.</i> | <i>&c.</i> | <i>&c.</i> |

Jam vero Series istæ sunt convergentes, adeoque singulæ sum-
 mari possunt per vulgarem Arithmetica; & obtinebuntur
 vel summæ accuratæ si possint, vel saltem approximatae, si non
 liceat, terminos multos adhibere.

Inveni-

Invenire summas Probabilitatum ad infinitum usque pergentium, quas Collusores habent ut victores evadant.

Sint Probabilitates omnes ipsius B ad infinitum, nempe
 $B' + B'' + B''' + B'''' + B^v + B^v' \&c. = \gamma$

Probabilitates ipsius C
 $C' + C'' + C''' + C'''' + C^v + C^v' \&c. = \alpha$

Probabilitates ipsius D
 $D' + D'' + D''' + D'''' + D^v + D^v' \&c. = \nu$

Probabilitates ipsius A
 $A' + A'' + A''' + A'''' + A^v + A^v' \&c. = x$

Scribantur autem in Scala perpendiculariter descendente, ad hunc modum.

$$\begin{aligned} B' &= B' \\ B'' &= B'' \\ B' &= \frac{1}{2} A'' + \frac{1}{4} A' \\ B''' &= \frac{1}{2} A''' + \frac{1}{4} A'' \\ B^v &= \frac{1}{2} A'''' + \frac{1}{4} A''' \\ B^v' &= \frac{1}{2} A^v + \frac{1}{4} A'''' \\ \text{Ergo } \gamma &= \frac{1}{4} + \frac{1}{2} x + \frac{1}{4} x. \end{aligned} \quad \text{Proinde } \gamma = \frac{1}{4} + \frac{1}{4} x.$$

Demonstratio.

Etenim prima columna perpendicularis $= \gamma$, ex Hypothesi
 Est vero $A' + A'' + A''' + A'''' + A^v \&c. = x$, ex hypothesi;
 Ergo $\frac{1}{2} A' + \frac{1}{2} A'' + \frac{1}{2} A''' + \frac{1}{2} A'''' + \frac{1}{2} A^v \&c. = \frac{1}{2} x$.
 Proinde $\frac{1}{2} A'' + \frac{1}{2} A''' + \frac{1}{2} A'''' + \frac{1}{2} A^v \&c. = \frac{1}{2} x - \frac{1}{2} A'$.
 Et $B' + B'' + \frac{1}{2} A' + \frac{1}{2} A'' + \frac{1}{2} A''' \&c. = \frac{1}{2} x - \frac{1}{2} A' + B' + B''$.

Sed

Sed $\frac{1}{2} A' = \bar{0}, B'' = 0$ & $B' = \frac{1}{4}$, ut patet ex Tabula:

Ergo secunda columna perpendicularis $= \frac{1}{4} + \frac{1}{2} x$.

Sed tertia columna perpendicularis $= \frac{1}{4} x$.

erit igitur $y = \frac{1}{4} + \frac{3}{4} x$.

Simili modo scribantur

$$C' = C'$$

$$C'' = C''$$

$$C''' = \frac{1}{2} B'' + \frac{1}{4} D'$$

$$C'''' = \frac{1}{2} B''' + \frac{1}{4} D'' \quad \text{hoc est } z = \frac{1}{2} y + \frac{1}{4} v$$

$$C^v = \frac{1}{2} B'''' + \frac{1}{4} D'''$$

$$C^v = \frac{1}{2} B^v + \frac{1}{4} D''''$$

$\&c.$

Ergo $z = \frac{1}{8} + \frac{1}{2} y - \frac{1}{8} + \frac{1}{4} v$.

Scribantur etiam

$$D' = D'$$

$$D'' = D''$$

$$D''' = \frac{1}{2} C'' + \frac{1}{4} B'$$

$$D'''' = \frac{1}{2} C''' + \frac{1}{4} B''$$

$$D^v = \frac{1}{2} C'''' + \frac{1}{4} B'''$$

$$D^v = \frac{1}{2} C^v + \frac{1}{4} B''''$$

$\&c.$

& pari Argumento patebit

$$v + \frac{1}{2} z + \frac{1}{4} y$$

Scribantur denique

$$A' = A'$$

$$A'' = A''$$

$$A''' = \frac{1}{2} D' + \frac{1}{4} C'$$

$$A'''' = \frac{1}{2} D'' + \frac{1}{4} C''$$

$$A^v = \frac{1}{2} D''' + \frac{1}{4} C'''$$

$$A^v = \frac{1}{2} D^v + \frac{1}{4} C''''$$

$\&c.$

Unde concludetur $x = \frac{1}{2} v + \frac{1}{4} z$

Refolutis autem quatuor istis æquationibus, reperietur

$$B' + B'' + B''' + B'''' \&c. = \gamma = \frac{56}{149}$$

$$C' + C'' + C''' + C'''' \&c. = z = \frac{36}{149}$$

$$D' + D'' + D''' + D'''' \&c. = v = \frac{32}{149}$$

$$A' + A'' + A''' + A'''' \&c. = x = \frac{25}{149}$$

Valoribus istis inventis, ponatur jam $\frac{56}{149} = b, \frac{36}{149} = c,$

$$\frac{32}{149} = d, \frac{25}{149} = a.$$

Iterum sit.

$$3 B' p + 4 B'' p + 5 B''' p + 6 B'''' p \&c. = p \gamma.$$

$$3 C' p + 4 C'' p + 5 C''' p + 6 C'''' p \&c. = p z.$$

$$3 D' p + 4 D'' p + 5 D''' p + 6 D'''' p \&c. = p v.$$

$$3 A' p + 4 A'' p + 5 A''' p + 6 A'''' p \&c. = p x.$$

$$3 B' = 3 B'$$

$$4 B'' = 4 B''$$

$$5 B''' = \frac{5}{2} A'' + \frac{5}{4} A'$$

$$6 B'''' = \frac{6}{2} A''' + \frac{6}{4} A''$$

$$7 B'' = \frac{7}{2} A'''' + \frac{7}{4} A'''$$

$$8 B'' = \frac{8}{2} A'' + \frac{8}{4} A''''$$

$$\text{Ergo } \gamma = \frac{3}{4} + \frac{1}{4} x + a$$

Etenim prima Columna perpendicularis = γ , ex Hypothefi:

$$3 B' + 4 B'' = \frac{3}{4}: \text{ Nam est } B' = \frac{1}{4}, \& B'' = 0.$$

$$3 A' + 4 A'' + 5 A''' \&c. = x \text{ ex Hypothefi.}$$

$$A' + A'' + A''' \&c. = a, \text{ ut repertum est.}$$

$$\text{Est igitur } 4 A' + 5 A'' + 6 A''' + 7 A'''' \&c. = x + a$$

$$\text{Et } \frac{4}{2} A + \frac{5}{2} A'' + \frac{6}{2} A'' + \frac{7}{2} A'''' \&c. = \frac{1}{2} x + \frac{1}{2} a.$$

Sed $A' = 0$

Ergo secunda Columna perpendicularis $= \frac{3}{4} + \frac{1}{2} x + \frac{1}{2} a$.

$$3 A' + 4 A'' + 5 A''' + 6 A'''' \&c. = x$$

$$2 A' + 2 A'' + 2 A''' + 2 A'''' \&c. = 2 a$$

Est igitur $5 A' + 6 A'' + 7 A''' + 8 A'''' \&c. = x + 2 a$.

Et $\frac{5}{4} A' + \frac{6}{4} A'' + \frac{7}{4} A''' + \frac{8}{4} A'''' \&c. = \frac{1}{4} x + \frac{1}{2} a$.

Est igitur tertia Columna perpendicularis $= \frac{1}{4} x + \frac{1}{2} a$.

Erit igitur $y = \frac{3}{4} + \frac{1}{2} x + \frac{1}{2} a + \frac{1}{4} x + \frac{1}{2} a$

sive $y = \frac{3}{4} + \frac{1}{2} x + a$, quod erat probandum.

$$3 C' = 3 C'$$

$$4 C'' = 4 C''$$

$$5 C''' = \frac{5}{2} B'' + \frac{5}{4} D'$$

$$6 C'''' = \frac{6}{2} B''' + \frac{6}{4} D''$$

$$7 C^v = \frac{7}{2} B'''' + \frac{7}{4} D'''$$

$$8 C^v' = \frac{8}{2} B^v + \frac{8}{4} D''''$$

$\&c.$

Ergo $z = \frac{1}{2} y + \frac{1}{2} b + \frac{1}{4} v + \frac{1}{2} d$.

Etenim prima Columna perpendicularis $= z$, ex Hypothesi.

$$3 C' + 4 C'' = \frac{1}{2}.$$

$$3 B' + 4 B'' + 5 B''' + 6 B'''' \&c. = y.$$

$$B' + B'' + B''' + B'''' \&c. = b.$$

Est igitur $4 B' + 5 B'' + 6 B''' + 7 B'''' \&c. = y + b$.

Sed $4 B' = 1$.

Ergo $5 B'' + 6 B''' + 7 B'''' \&c. = y + b - 1$.

$$\frac{5}{2} B'' + \frac{6}{2} B''' + \frac{7}{2} B'''' \&c. = \frac{1}{2} y + \frac{1}{2} b - \frac{1}{2}.$$

Ergo secunda Columna perpendicularis $= \frac{1}{2} + \frac{1}{2} y + \frac{1}{2} b - \frac{1}{2}$
 $= \frac{1}{2} y + \frac{1}{2} b$.

Iterum, $3 D' + 4 D'' + 5 D''' + 6 D'''' \&c. = v$

$$2 D' + 2 D'' + 2 D''' + 2 D'''' \&c. = 2 d$$

Est igitur $5 D' + 6 D'' + 7 D''' + 8 D'''' \&c. = v + 2 d$.

$$\text{Et } \frac{5}{4} D' + \frac{6}{4} D'' + \frac{7}{4} D''' + \frac{8}{4} D'''' \&c. = \frac{1}{4} v + \frac{1}{2} d.$$

Ergo tertia Columna perpendicularis $= \frac{1}{4} v + \frac{1}{2} d$

Est igitur $z = \frac{1}{2} y + \frac{1}{2} b + \frac{1}{4} v + \frac{1}{2} d$, quod erat probandum.

Eodem prorsus ordine scribantur.

$$\begin{aligned} 3 D' &= 3 D' \\ 4 D'' &= 4 D'' \\ 5 D''' &= \frac{5}{2} C'' + \frac{5}{4} B' \\ 6 D'''' &= \frac{6}{2} C''' + \frac{6}{4} B'' \\ 7 D^V &= \frac{7}{2} C'''' + \frac{7}{4} B''' \\ 8 D^{VI} &= \frac{8}{2} C^V + \frac{8}{4} B'''' \end{aligned}$$

&c.

$$\text{Unde } v = \frac{1}{2} z + \frac{1}{2} c + \frac{1}{4} y + \frac{1}{2} b.$$

$$\begin{aligned} 3 A' &= 3 A' \\ 4 A'' &= 4 A'' \\ 5 A''' &= \frac{5}{2} D'' + \frac{5}{4} C' \\ 6 A'''' &= \frac{6}{2} D''' + \frac{6}{4} C'' \\ 7 A^V &= \frac{7}{2} D'''' + \frac{7}{4} C''' \\ 8 A^{VI} &= \frac{8}{2} D^V + \frac{8}{4} C'''' \end{aligned}$$

&c.

$$\text{Et } x = \frac{1}{2} v + \frac{1}{2} d + \frac{1}{4} z + \frac{1}{2} c.$$

Quæ quidem Conclusiones eodem modo demonstrantur ac superiores.

Solutis autem quatuor istis æquationibus, elicitur

$$y = \frac{45536}{149|^2}, z = \frac{38724}{149|^2}, v = \frac{37600}{149|^2}, x = \frac{33547}{149|^2} = \frac{33547}{22201}$$

Ergo, si velint B, C, D, A vendere Spectatori cuidam R summas quas singuli obtinere sperant, æquum erit ut emptor R pendat

$$\begin{aligned} \text{ipsi } B \quad 4 \times \frac{56}{149} + \frac{45536}{22201} p, \quad \text{ipsi } C \quad 4 \times \frac{36}{149} + \frac{38724}{22201} p. \\ \text{ipsi } D \quad 4 \times \frac{32}{149} + \frac{37600}{22201} p, \quad \text{ipsi } A \quad 4 \times \frac{25}{149} + \frac{33547}{22201} p. \end{aligned}$$

Invenire Probabilitates quas habent B, C, D, A , ut mulctentur, dato ludorum numero.

Si Ludi duo tantum sint, erunt hoc modo.

$$\left. \begin{array}{l} \frac{BA}{CB} \quad \frac{BA}{BC} \end{array} \right\} \text{Unde patet } B \text{ vel } C \text{ necessario mulctari.}$$

Si Ludi tres fuerint, hoc modo se res habet.

$$\left. \begin{array}{l} \frac{BA}{CB} \quad \frac{BA}{CB} \quad \frac{BA}{BC} \quad \frac{BA}{BC} \\ \frac{DC}{CD} \quad \frac{CD}{DB} \quad \frac{DB}{BD} \quad \frac{BD}{BC} \end{array} \right\} \text{Hinc patet } C, \text{ vel } D \text{ vel } B \text{ necessario mulctari.}$$

Si vero quatuor Ludi fuerint:

| | | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|---|
| \overline{BA} | \overline{BA} | \overline{BA} | \overline{BA} | \overline{BA} | \overline{BA} | } Debet igitur A triplici modo, D duplici, C simplici, mulctari. |
| \overline{CB} | \overline{CB} | \overline{CB} | \overline{CB} | \overline{BC} | \overline{BC} | |
| \overline{DC} | \overline{DC} | \overline{CD} | \overline{CD} | \overline{DB} | \overline{DB} | |
| \overline{AD} | \overline{DA} | \overline{AC} | \overline{CA} | \overline{AD} | \overline{DA} | |
| | | | | | | |

Et sic de cæteris. Ex quibus manifesta est Compositio Tabulæ subjunctæ Probabilitatum quas B, C, D, A habent ut mulctentur, dato ludorum numero.

| | Num Lud. | B | C | D | A |
|------|-------------|----------------|----------------|----------------|----------------|
| ' | 2 | $\frac{1}{2}$ | $\frac{1}{2}$ | | |
| " | 3 | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{2}{4}$ | |
| ''' | 4 | | $\frac{1}{8}$ | $\frac{2}{8}$ | $\frac{3}{8}$ |
| '''' | 5 | $\frac{3}{16}$ | $\frac{2}{16}$ | $\frac{2}{16}$ | $\frac{3}{16}$ |
| v | 6 | $\frac{6}{32}$ | $\frac{5}{32}$ | $\frac{2}{32}$ | $\frac{3}{32}$ |
| v' | 7 | $\frac{6}{64}$ | $\frac{8}{64}$ | $\frac{8}{64}$ | $\frac{4}{64}$ |
| | | $\&c.$ | | | |

Sint autem y, z, v, x summæ omnium Probabilitatum quas B, C, D, A habent respective ut mulctentur.

Scribantur eodem ordine ac in præcedentibus.

$$B' = B'$$

$$C' = C'$$

$$B'' = B''$$

$$C'' = C''$$

$$B''' = \frac{1}{2} A' + \frac{1}{4} A'$$

$$C''' = \frac{1}{2} B'' + \frac{1}{4} D'$$

$$B'''' = \frac{1}{2} A'' + \frac{1}{4} A''$$

$$C'''' = \frac{1}{2} B''' + \frac{1}{4} D''$$

$$B^v = \frac{1}{2} A''' + \frac{1}{4} A'''$$

$$C^v = \frac{1}{2} B'''' + \frac{1}{4} D'''$$

$$B^{v'} = \frac{1}{2} A^v + \frac{1}{4} A^{v'}$$

$$C^{v'} = \frac{1}{2} B^{v''} + \frac{1}{4} D^{v''}$$

$\&c.$

$\&c.$

$$\text{Ergo } y = \frac{3}{4} + \frac{1}{2} x + \frac{1}{4} x.$$

$$\text{Ergo } z = \frac{1}{2} + \frac{1}{2} y + \frac{1}{4} v.$$

Scri-

Scribantur deinde

$$D' = D'$$

$$D'' = D''$$

$$D''' = \frac{1}{2} C'' + \frac{1}{4} B'$$

$$D'''' = \frac{1}{2} C''' + \frac{1}{4} B''$$

$$D^v = \frac{1}{2} C'''' + \frac{1}{4} B'''$$

$$D^{vi} = \frac{1}{2} C^v + \frac{1}{4} B''''$$

Ecce.

$$\text{Ergo } v = \frac{1}{4} + \frac{1}{2} z + \frac{1}{4} y.$$

$$A' = A'$$

$$A'' = A''$$

$$A''' = \frac{1}{2} D' + \frac{1}{4} C'$$

$$A'''' = \frac{1}{2} D'' + \frac{1}{4} C''$$

$$A^v = \frac{1}{2} D''' + \frac{1}{4} C'''$$

$$A^{vi} = \frac{1}{2} D^v + \frac{1}{4} C''''$$

Ecce

$$\text{Ergo } x = \frac{1}{2} v + \frac{1}{4} z.$$

Resolutis autem quatuor istis æquationibus, invenietur

$$y = \frac{243}{249} \quad z = \frac{252}{149} \quad v = \frac{224}{149} \quad \& x = \frac{175}{149}$$

Ergo si velit Spectator aliquis S multas omnes sustinere, æquum erit ut ipsi S

$$B \text{ tradat } \frac{243}{149} p \quad C \quad \frac{252}{149} p \quad D \quad \frac{224}{149} p \quad \& A \quad \frac{175}{149} p.$$

Sublatis itaque summis probabilitatum quas singuli Collusores habent ut mulctentur, è summis expectationum quas habent iidem si victores abeant, restabunt sortes eorum respective : nempe

$$B \text{ recipit ab } R \quad \frac{4 \times 56}{149} + \frac{45536}{22201} p$$

$$B \text{ tradit ipsi } S \quad \frac{243}{149} p$$

$$\text{Ergo ipsi } B \text{ superest } \frac{224}{149} + \frac{9329}{22201} p$$

Sed B deposuerat priusquam ludus inciperetur.

$$\text{Ergo } B \text{ lucratur } \frac{75}{149} + \frac{9329}{22201} p.$$

(156)

$$C \text{ recipit ab } R \quad \frac{4 \times 36}{149} + \frac{38724}{22201} p$$

$$C \text{ tradit ipsi } S \quad \frac{252}{149} p$$

$$\text{Ergo ipsi } C \text{ superest } \frac{144}{149} + \frac{1176}{22201} p$$

Sed C deposuerat 1.

$$\text{Ergo } C \text{ lucratur } - \frac{5}{149} + \frac{1176}{22201} p.$$

$$D \text{ recipit ab } R \quad \frac{4 \times 32}{149} + \frac{37600}{22201} p$$

$$D \text{ tradit ipsi } S \quad \frac{224}{149} p$$

$$\text{Ergo ipsi } D \text{ superest } \frac{128}{149} + \frac{4224}{22201} p$$

Sed D deposuerat 1.

$$\text{Ergo } D \text{ lucratur } - \frac{21}{149} + \frac{4224}{22201} p.$$

$$A \text{ recipit ab } R \quad \frac{4 \times 25}{149} + \frac{33547}{22201} p$$

$$A \text{ tradit ipsi } S \quad \frac{175}{149} p$$

$$\text{Ergo ipsi } A \text{ superest } \frac{100}{149} + \frac{7472}{22201} p$$

Sed A deposuerat 1 + p , nempe 1 priusquam ludus inchoaretur, & p postquam semel victus fuerat à B :

$$\text{Ergo } A \text{ lucratur } - \frac{49}{149} - \frac{14719}{22201} p.$$

Lucrum

$$\text{Lucrum ipsius } B = + \frac{75}{149} + \frac{9329}{22201} p$$

$$\text{ipsius } C = - \frac{5}{149} + \frac{1176}{22201} p$$

$$\text{ipsius } D = - \frac{21}{149} + \frac{4224}{22201} p$$

$$\text{ipsius } A = - \frac{49}{149} - \frac{14729}{22201} p$$

$$\text{Summa Lucrorum} = \quad 0 \quad 0$$

$$\text{Summa autem lucrorum ipsorum } B \text{ \& } A = \frac{26}{149} - \frac{5400}{22201} p ;$$

sed posueramus *B* vicisse ipsum *A* semel, priusquam Collusores pacta inirerent cum *R* & *S*. Priusquam vero ludus inchoaretur, *A* poterat æqua sorte expectare ut vinceret ipsum *B*,

adeoque summa lucrorum $\frac{26}{149} - \frac{5400}{22201}$ in duas partes æqua-

les est dividenda, ita ut utriusque lucrum censendum sit $\frac{13}{149} - \frac{2700}{22201} p$.

$$\text{Ponatur } \frac{13}{149} - \frac{2700}{22201} p = 0, \text{ \& erit } p = \frac{1937}{2700}.$$

Ergo si sit multa *p* ad summam quam singuli deponunt ut 1937 ad 2700, *A* & *B* nihil lucrantur, nihil perdunt. Verum

hoc in Casu *C* lucratur $\frac{1}{225}$, quam *D* perdit.

Coroll. 1. Spectator *R*, priusquam ludus inchoetur, id suscipere in se poterit, ut summam 4 de qua Collusores contendunt, & multas omnes pendat, si sibi initio in manus darentur 4 + 7 *p*.

Coroll. 2. Si dexteritates Collusorum sint in ratione data, sortes Collusorum eadem ratiocinatione determinabuntur.

Coroll. 3. Si Series aliqua ita sit constituta, ut continuò decrescat, & terminus quivis ad præcedentes quoslibet habeat rationes datas, five easdem five diversas, series ista accurate summabitur. Insuper si termini omnes hujus Series multiplicentur per terminos progressionis Arithmeticæ, singuli per singulos, Series nova resultans accurate summabitur.

Coroll. 4. Si sint Series plures collaterales, ita relatæ ut terminus quilibet cujusque Series ad præcedentes quoslibet aliarum Serierum habeat rationes datas, five easdem five diversas, ita ut Series istæ collaterales se decussent data qualibet lege constanti, Series istæ accurate summabuntur. Insuper si termini omnes harum Serierum multiplicentur ordinatim per terminos Progressionis Arithmeticæ, singuli per singulos, Series novæ ex hac multiplicatione resultantes etiamnum accurate summabuntur.

Clavis ad Problema generale.

Si sint Collusores quotcunque *v. g.* Sex, *B, C, D, E, F, A* & Probabilitates quas habent ut victores evadant, five ut mulctentur, dato Ludorum numero, denotentur respective *B, C, D, E, F & A*; & Probabilitates dato Ludorum numero his proximo & minori competentes, per *B_{II}, C_{II}, D_{II}, E_{II}, F_{II}, A_{II}*; & Probabilitates dato Ludorum numero his itidem novissimis proximo & minori competentes, per *B_{III}, C_{III}, D_{III}, E_{III}, F_{III}, A_{III}*, & sic deinceps; erit semper,

$$\begin{aligned} B &= \frac{1}{2}A_{II} + \frac{1}{4}A_{III} + \frac{1}{8}A_{III} + \frac{1}{16}A_V \\ C &= \frac{1}{2}B_{II} + \frac{1}{4}F_{III} + \frac{1}{8}E_{III} + \frac{1}{16}D_V \\ D &= \frac{1}{2}C_{II} + \frac{1}{4}B_{III} + \frac{1}{8}F_{III} + \frac{1}{16}E_V \\ E &= \frac{1}{2}D_{II} + \frac{1}{4}C_{III} + \frac{1}{8}B_{III} + \frac{1}{16}F_V \\ F &= \frac{1}{2}E_{II} + \frac{1}{4}D_{III} + \frac{1}{8}C_{III} + \frac{1}{16}B_V \\ A &= \frac{1}{2}F_{II} + \frac{1}{4}E_{III} + \frac{1}{8}D_{III} + \frac{1}{16}C_V \end{aligned}$$

Et fiat semper retrogressus ordinatim ad tot literas minus duobus quot sunt Collusores, omittaturque semper litera *A*, prima æquatione excepta, ubi litera *A* terminos omnes præter primam occupat.